Statistical testing in latent variable models

A small recap

What are latent variables?

Latent variables are not directly observed but are inferred from other variables that are observed. An example of a latent variable could be quality of life - an unobserved variable that could be measured from many observed variables such as wealth, employment, physical and mental health.

What are latent variable models?

A latent variable model is a statistical model that relates a set of observable variables to a set of latent variables.

What is statistical testing?

Statistical testing focuses on deciding whether there is enough evidence (data) to make a quantitative decision about a process - i.e. to reject a hypothesis supporting it. However, not not rejecting (while not proving either) may be a good result.

Statistical testing in latent variable models

Statistical testing in latent variable models can be used for two main purposes:

- testing hypotheses about the relationships between variables
- evaluating the fitness of a statistical model

Whether it is (1) or (2), there are many commonly used tests or methods. However, for (1) there are also additional procedures/measures such as evaluating the path coefficients of a given statistical model or factor loadings.

Evaluating the fitness of a statistical model

Though several varying opinions exist, Kline (2010) recommends reporting the chi-squared test, the root mean square error of approximation (RMSEA), the comparative fit index (CFI), and the standardized root mean square residual (SRMR).
via wikipedia.com: Kline, R. B. (2010). Principles and practice of structural equation modeling (3rd ed.). New York, New York: Guilford Press.

1. Chi-Square Test of Model Fit

   This is a test highly dependent on the sample size - it uses an approximation that becomes exact when in the limit as the sample size grows to infinity (as many other statistical tests).

$$X^2 = \sum \frac{(\text{Observed value - Expected value})^2}{\text{Expected value}}$$

   It tests whether the covariance matrix derived from the model represents the population covariance. A non-significant chi-square value suggests a good fit. ***It was introduced by Karl Pearson in 1900.***

2. Comparative Fit Index (CFI) or Confirmatory Factor Analysis (CFA)

   In comparative factor analysis several tests are used to assess how well the model fits the data. Remember, that a “good model fit” only indicates that the model is plausible, not that the model is correct or ‘good’.

   Usually, the CFI compares the fit of a target model to the fit of an independent, or null, model, while adjusting for the issues of sample size inherent in the chi-squared test of model fit. ***The Comparative Fit Index (CFI) was developed by Bentler in 1990.***

3. Root Mean Square Error of Approximation (RMSEA)

$$\sqrt{\frac{\chi 2 - \mathrm{df}}{df\,(n-1)}}$$

   Where, n is the sample size, df is the number of degrees of freedom and χ2 is the chi-square statistic of the model. The degrees of freedom mean the maximum number of logically independent values, which may vary in a data sample. **It was introduced by Steiger, J. H., & Lind, J. C. in 1980.**

4. Standardized Root Mean Square Residual (SRMR)

   The SRMR is an absolute measure of model fit and is defined as the difference between the observed correlation and the model inferred correlation matrix.

$$\mathrm{SRMR} = \sqrt{\left\{2\sum_{i=1}^{p}\sum_{j=1}^{i}\left[\left(s_{ij}-\hat{\sigma}_{ij}\right)/\left(s_{ii}s_{jj}\right)\right]^{2}\right\}/\,p(p+1)}$$

Note. $T_T = T$ statistic for the target model; $df_T = df$ for the target model; $T_B = T$ statistic for the baseline model; $df_B = df$ for the baseline model; p = number of observed variables; s_{ij} = observed covariances; $\hat{\sigma}_{ij}$ = reproduced covariances; s_{ii} and s_{jj} are the observed standard deviations; TLI =

Hu and Bentler (1999)